LTL Properties

Now you know:

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- how it can be used for system analysis

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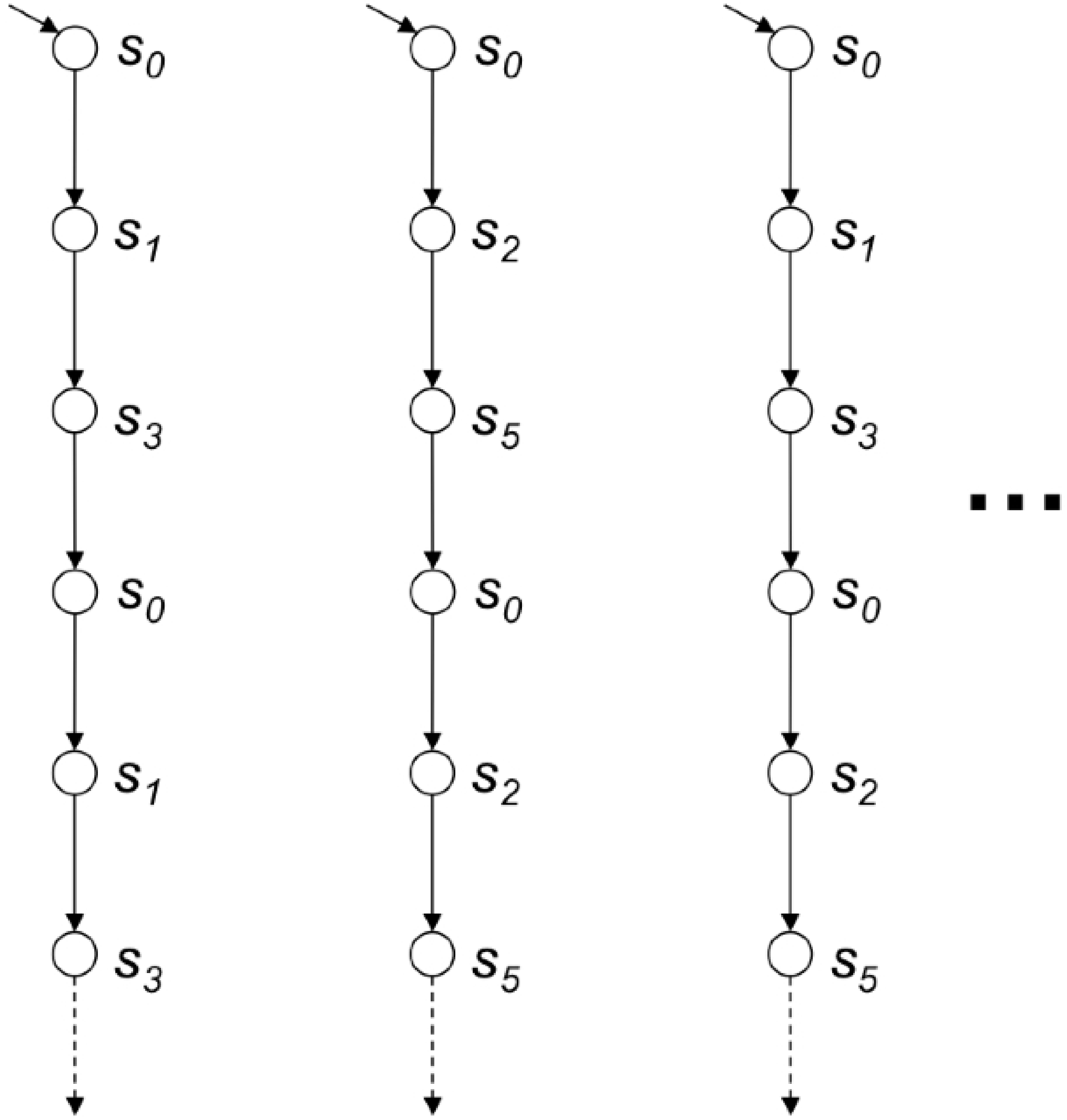
Let's see LTL logics to express properties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Logics to express linear time properties

- LTL = Linear-time Temporal Logic
- Syntax: let AP be a set of atomic propositions.
  - $a \in AP$ is an LTL formula.
  - If ϕ_1 and ϕ_2 are LTL formulae then so are
    $\neg\phi_1$ $\phi_1 \wedge \phi_2$ $\mathrm{X}\ \phi_2$ $\phi_1\ \mathrm{U}\ \phi_2$
    where X stands for "next" and U for "until"
  - Two famous short-cuts: $F\phi \equiv true\ \mathrm{U}\ \phi$ and $G\phi \equiv \neg F \neg \phi$

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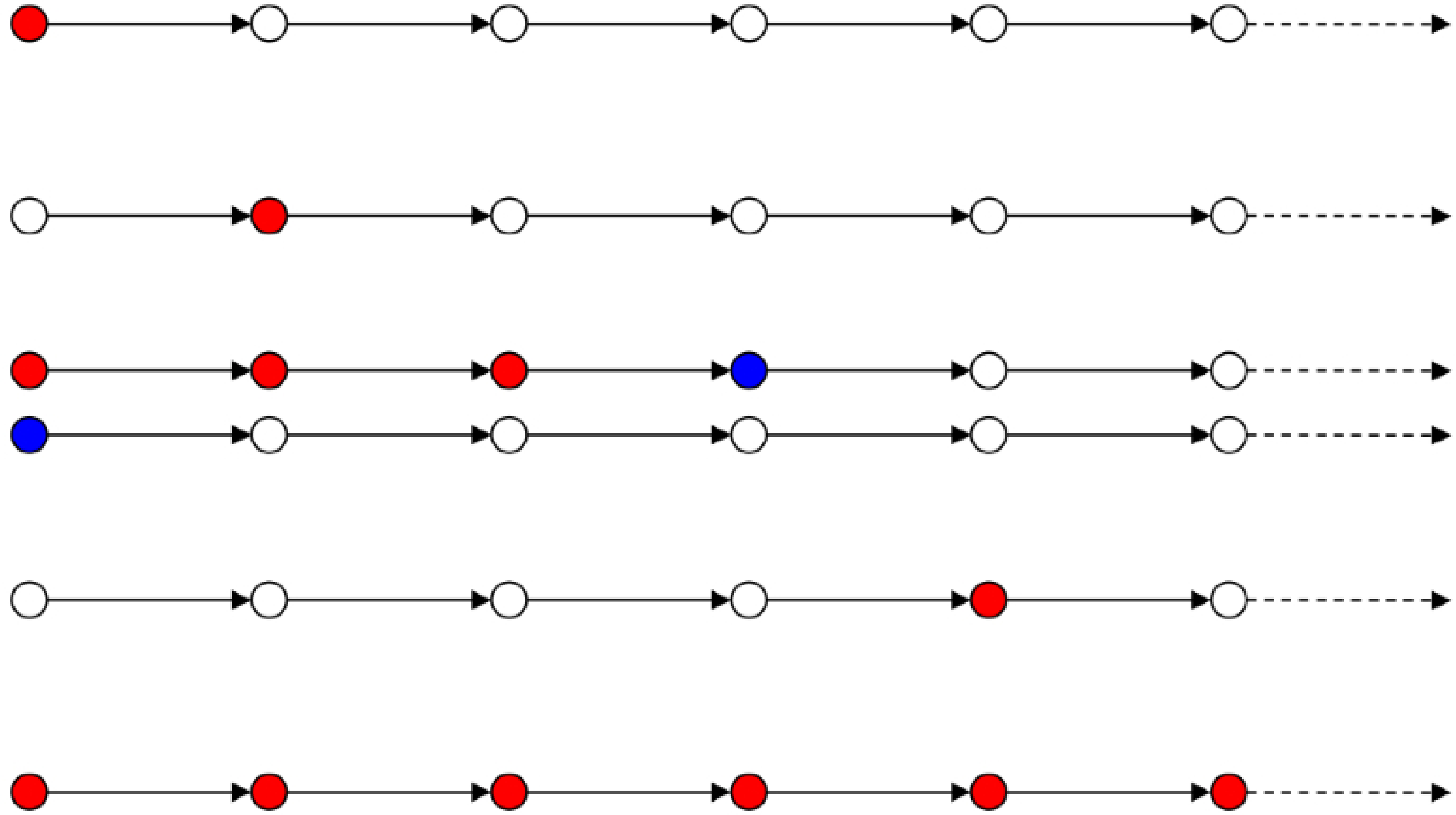
  - Two famous short-cuts: $F\phi \equiv true\,\mathsf{U}\,\phi$ and $G\phi \equiv \neg F \neg\phi$
- To each LTL formula ϕ, we associate a language $\mathcal{L}(\phi)$ of ω-words over 2^{AP} (i.e., we have $\mathcal{L}(\phi) \subseteq (2^{AP})^\omega$). Let $\sigma \in (2^{AP})^\omega$:

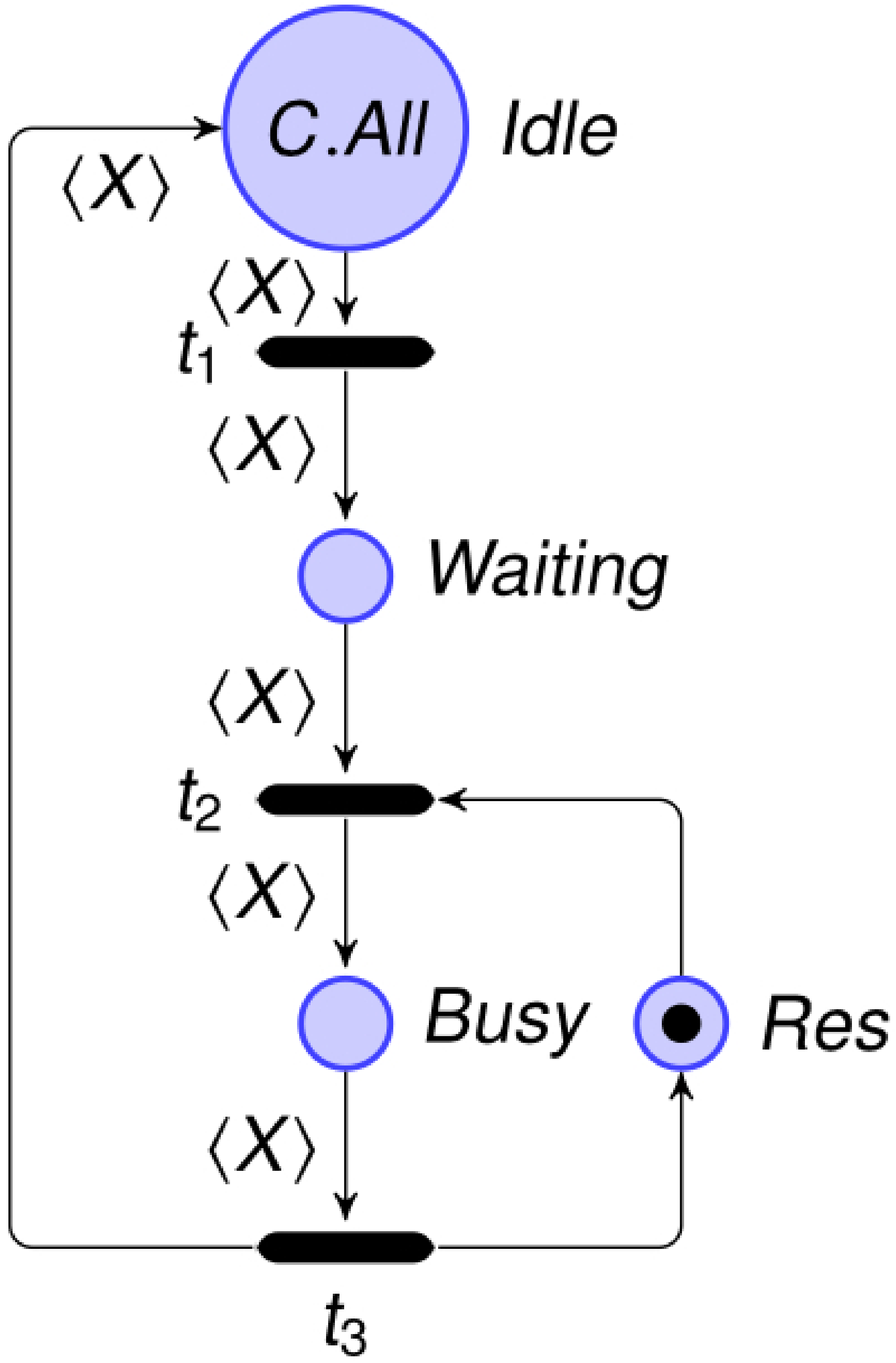
$$
\begin{aligned}
\sigma \in \mathcal{L}(a) &\Leftrightarrow a \in \sigma(0)\\
\sigma \in \mathcal{L}(\neg\phi) &\Leftrightarrow \neg\sigma \in \mathcal{L}(\phi)\\
\sigma \in \mathcal{L}(\phi_1 \wedge \phi_2) &\Leftrightarrow \sigma \in \mathcal{L}(\phi_1) \cap \mathcal{L}(\phi_2)\\
\sigma \in \mathcal{L}(\mathsf{X}\phi) &\Leftrightarrow \sigma^1 \in \mathcal{L}(\phi)\\
\sigma \in \mathcal{L}(\phi_1 \,\mathsf{U}\, \phi_2) &\Leftrightarrow \exists i : \sigma^i \in \mathcal{L}(\phi_2) \wedge \forall k < i, \sigma^k \in \mathcal{L}(\phi_1)
\end{aligned}
$$

Illustration of the LTL semantics

- a
- $X\ \phi$
- $\phi_1\ U\ \phi_2$
- $F\ \phi$
- $G\ \phi$



Examples of LTL formulae



- Atomic propositions:
  - $p(c)$, where, $p \in P \setminus \{Res\}$ and $c \in C$
  - $t(c)$, where, $t \in T$ and $c \in C$
- $\mathsf{G}\neg(Busy(c_1)\ \&\ Busy(c_2))$: it always holds that c_1 and c_2 do not appear together in critical section (place *Busy*).
- $\mathsf{G}(Waiting(c_3) \Rightarrow \mathsf{F}(Busy(c_3)))$: whenever c_3 requests to enter its critical section, it will eventually succeed.

Conclusion

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Let's see CTL logics to express additional properties (next sequence)