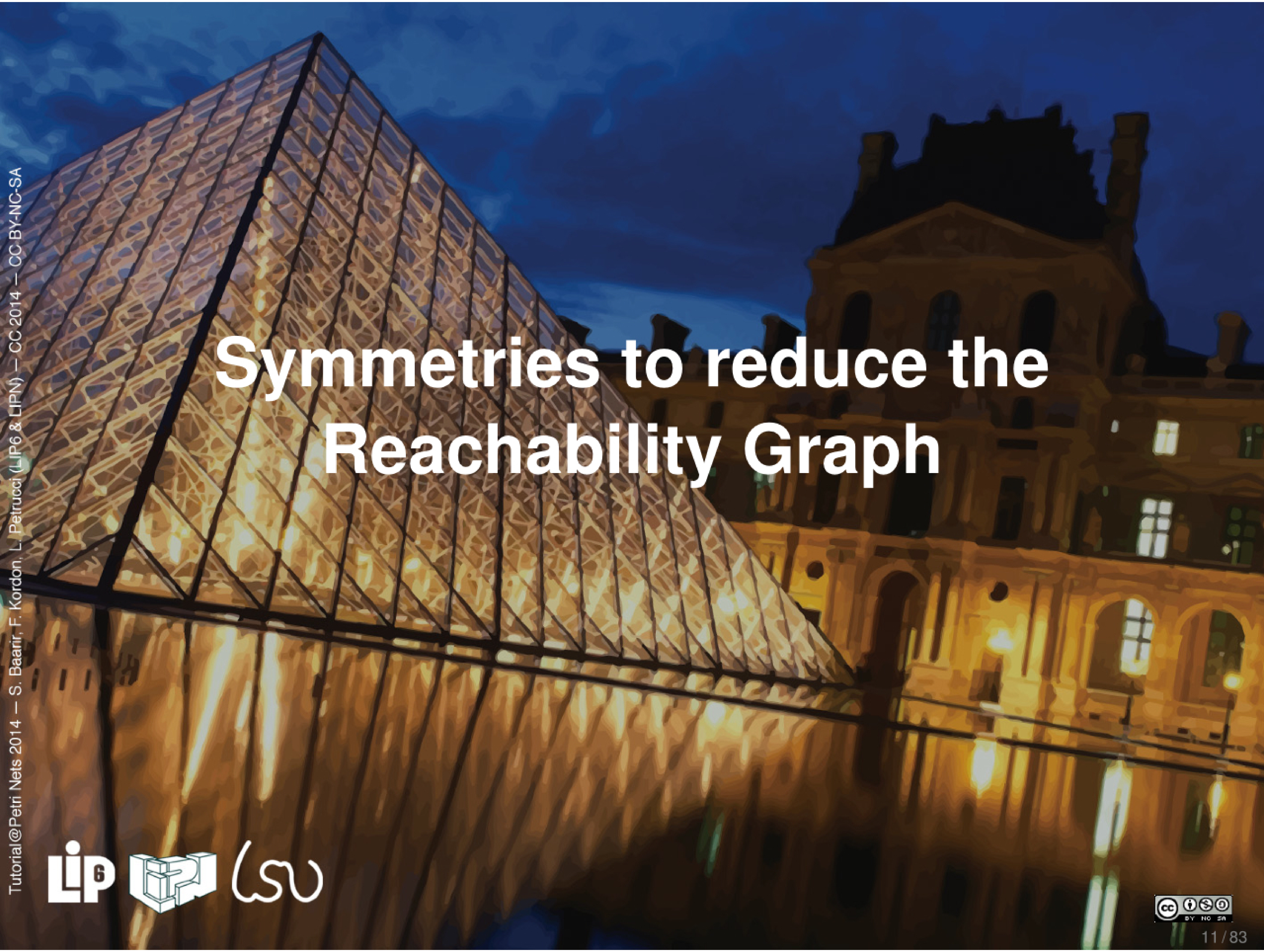

Symmetries to reduce the Reachability Graph

Introduction

Now, you:

- know Symmetric Nets with their syntax and semantics,
- have an intuitive idea about the notion of symmetries in SNs.

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Let's now study, formally, these symmetries and their usage for the construction of a reduced reachability graph.

Symmetries and SNs

- Consider a net $N = \langle P, T, C, W^-, W^+, M_0 \rangle$.
- Consider the set $S = \{\langle s_1, \ldots, s_n \rangle | s_i \in S_i\}$, where,
  1. With each unordered class C_i, we associate the (total) permutation group S_i.
  2. With each ordered class C_i, we associate the (total) rotation group S_i.

  We call S the set of symmetries of a SN.
- Useful properties: let C_i be a colour class and $f_i : C(t) \rightarrow Bag(C_i)$ (a colour function) and s_i the associated symmetry
  1. $f_i = X_i^j \Rightarrow s_i \circ f_i = f_i \circ s_i, \forall s_i \in S_i$.
  2. $f_i = C_i.All \Rightarrow s_i \circ f_i = f_i \circ s_i = C_i.All, \forall s_i \in S_i$.
  3. $f_i = X_i^j{++} \Rightarrow r_i \circ f_i = f_i \circ r_i, \forall r_i \in S_i$. (when C_i is ordered).

Markings Equivalence and Markings Classes

- Markings equivalence ($\equiv_S$):

$$M \equiv_S M' \Leftrightarrow \exists s \in S, M' = s.M$$

- For each marking M, we define its marking class (orbit) with respect to S, $[M]_S$:

$$[M]_S = \{M \mid \exists s \in S, M = s.M\}$$

Enabling Equivalence

(t, c_t) **is enabled in a marking** M

$$\Updownarrow$$

$(t, s.c_t)$ **is enabled in the marking** $s.M$

- (t, c_t) is enabled in a marking M
  $\Leftrightarrow \; M(p) \geq W^{-}(p,t)(c_t)$
  $\Leftrightarrow \; \forall c \in C(p), M(p)(c) \geq W^{-}(p,t)(c_t)(c)$
  $\Leftrightarrow \; \forall c \in C(p), s.M(p)(s.c) \geq s.W^{-}(p,t)(c_t)(s.c)$
  Since, $s.W^{-}(p,t)(c_t) = W^{-}(p,t)(s.c_t)$, then
  $\Leftrightarrow \; \forall c \in C(p), s.M(p)(s.c) \geq W^{-}(p,t)(s.c_t)(s.c)$
  $\Leftrightarrow \; \forall c \in C(p), s.M(p)(c) \geq s.W^{-}(p,t)(c_t)(c)$
  $\Leftrightarrow \; (t, s.c_t)$ is enabled in a marking $s.M$

Firing Equivalence

$$M \xrightarrow{(t,c_t)} M' \Leftrightarrow s.M \xrightarrow{(t,s.c_t)} s.M'$$

- $M \xrightarrow{(t,c_t)} M'$
  $\Leftrightarrow \; M(p) = M(p) - W^{-}(p,t)(ct) + W^{+}(p,t)(c_t)$
  $\Leftrightarrow \; s.M(p) = s.M(p) - s.W^{-}(p,t)(c_t) + s.W^{+}(p,t)(c_t)$
  Since, $s.W^{-}(p,t)(c_t) = W^{-}(p,t)(s.c_t)$, and
  $s.W^{+}(p,t)(c_t) = W^{+}(p,t)(s.c_t)$, then
  $\Leftrightarrow \; s.M(p) = s.M(p) - W^{-}(p,t)(s.c_t) + W^{+}(p,t)(s.c_t)$
  $\Leftrightarrow \; s.M \xrightarrow{(t,s.c_t)} s.M'$

Conclusion

At this stage, you know:

- Symmetric Nets with their syntax and semantics,
- the formal definition definition of symmetries in SNs,
- The formal definition of markings and firings equivalences.

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At this stage, you know:

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How to use this notions to derive, automatically, a quotient reachability graph.